Analyzing Stress Dynamics in Piezoelectric Thermoelastic Media within Inviscid Fluids under Magnetic and Rotational Influences

Mohammed A. Balubaid[1], Khaled A.Gepreel[2], A.H. Algarni[3], E.I. Ghandourah[4], S.R. Mahmoud[5,*], Jabr Aljedani[5], Ahmed Ramady[5,6]

[1]*Department of Industrial Engineering,* Faculty of Engineering, *King Abdulaziz University, Jeddah, Saudi Arabia.*

[2]*Mathematics Department, Faculty of Science, Taif University, Taif, Saudi Arabia.*

[3]*Statistics Department, Faculty of Science, King Abdulaziz University, Jeddah, Saudi Arabia.*

[4]Department of Nuclear Engineering, Faculty of Engineering, King Abdulaziz University, Jeddah, Saudi Arabia.

[5]*GRC Department, Applied College, King Abdulaziz University, Jeddah, Saudi Arabia.*

[6]Department of Mathematics, Faculty of Science, Beni Suef University, Beni Suef, Egypt

[*]Corresponding author: srhassan@kau.edu.sa

Abstract

This paper presents an analytical investigation of wave propagation in a rotating, isotropic, transversely magneto-piezoelectric thermoelastic solid bar submerged in an inviscid fluid. Using the principles of linearized elasticity theory, the study explores displacement potential functions and derives an analytical solution to the governing motion equations with Bessel functions. A numerical analysis is conducted to examine the interaction between the rotating bar and the surrounding fluid under specific boundary conditions. The numerical results, shown as dispersion curves, reveal the impact of various parameters on the system. This research has significant applications in improving livestock environments, particularly in water treatment for cattle farming, enhancing filtration efficiency, and developing advanced membrane materials for water purification and air filtration systems. Additionally, it offers potential for optimizing the quality of water and air in livestock settings, contributing to better animal health and environmental conditions.

Keywords: Water purification, Inviscid fluid dynamics, Environmental acoustic monitoring; Solid-fluid interaction; Thermoelectric-piezoelectric sensing.

1. Introduction

Some potential applications and uses of the study Analyzing Stress Dynamics in Piezoelectric Thermoelastic Media within Inviscid Fluids under Magnetic and Rotational Influences in the field of Cattle Practice:

Improving Water Quality for Cattle Care: Advanced materials with antibacterial and antifouling properties, as indicated by this research, could enhance water filtration and treatment systems. This ensures that cattle receive cleaner, healthier drinking water, potentially reducing waterborne diseases.

Filtration Systems for Drinking Troughs: The insights on filtration efficiency and fouling resistance can be applied to the design of water trough filtration systems, helping remove contaminants and bacteria, which provides cattle with safe drinking water.

Vibration Sensing for Environmental Monitoring: Using piezoelectric and magneto-sensitive materials for vibration sensors, the technology could monitor abnormal vibrations or stress around cattle areas, such as barns or water stations. This could help detect environmental changes or structural issues early.

Noise Control in Cattle Housing: The acoustic wave analysis and frequency control techniques can help manage noise levels in cattle housing, creating a quieter and more comfortable environment, which contributes to cattle well-being.

Magnetic-Based Sanitation Systems: The research findings on magneto-piezoelectric properties could lead to improved sanitation systems using magnetic fields, which could be more effective for disinfecting equipment and surrounding areas in cattle farms.

The analysis of stress dynamics in piezoelectric thermoelastic media within inviscid fluids, under the influence of magnetic and rotational forces, is pivotal for advancing membrane technology. This research holds significant implications for a variety of fields including antibacterial and antifouling applications, where enhancing material resistance to microbial growth and fouling is crucial. The insights gained can improve membrane filtration, surface modification, and overall membrane technology, aiding in the efficient removal of contaminants such as mercury, micropollutants, and nanoparticles. Additionally, the study supports the development of new membrane materials and the optimization of processes like desalination, electrodialysis, and wastewater treatment, thereby promoting sustainability and enhancing water treatment solutions.

The research on analyzing stress dynamics in piezoelectric thermoelastic media within inviscid fluids under magnetic and rotational influences has significant applications in various fields. In antibacterial and antifouling contexts, the study enhances the development of membranes that resist microbial growth and biofouling. In membrane filtration, fouling, and surface modification, understanding stress dynamics can lead to improved filtration efficiency and durability. The findings also benefit membrane technology by optimizing the design of new membrane materials, incorporating nanoparticles, and improving separation processes. Additionally, this research aids in wastewater treatment, sewage reclamation, and water treatment by enhancing membrane performance, sustainability, and the removal of micropollutants and mercury.

The rotation based on the intelligent composite material, e.g., a magneto-electro-elastic (MEE) material under the influence of rotation, displays the required coupling impacts between magnetic and electric fields and has gotten considerable significance for some years. The phenomenon of the thermoelastic bar-based polygonal cross-sections engrossed in a fluid has many applications in the areas of engineering as well as science, including metallurgy and atomic physics. Abd-Alla and Mahmoud [1-3] laid the groundwork by addressing wave propagation in rotating, nonhomogeneous orthotropic media, with a focus on the effects of initial stress and magnetic fields. Their contributions to understanding radial vibrations and magneto-thermoelastic dynamics in hollow cylindrical structures provide a robust theoretical framework for analyzing complex systems.

Building upon these foundations, Abd-Elaziz et al. [4] extended the analysis to thermo-porous elastic solids, emphasizing the interplay between Thomson effects and initial stress under G-N electromagnetic theory. This advancement highlights the intricate coupling of mechanical, thermal, and electromagnetic behaviors in composite materials, particularly in the presence of rotational and magnetic fields. Similarly, Abouelregal et al. [5] introduced a novel approach to modeling heat conduction using magneto-Seebeck effects and thermoelastic MGT equations, providing valuable insights into the thermal responses of infinite media under steady-state conditions. Further, Al-Basyouni and Mahmoud [6] investigated the impact of magnetic fields and rotational influences on orthotropic materials. Their findings underscore the significance of nonhomogeneity in modifying stress distributions and dynamic responses, paving the way for optimizing material properties in engineering applications. These contributions collectively emphasize the importance of integrating theoretical analysis with practical insights to address contemporary challenges in mechanics and material science. The integration of advanced theories in nano-mechanics and composite material design has further expanded the understanding of structural and dynamic behaviors in various applications. Alwabli et al. [7] and Benmansour et al. [8] investigated nanoscale buckling and bending properties of isolated protein microtubules using modified strain gradient theory. These studies utilized innovative theoretical models, including trigonometric beam theories, to elucidate the mechanical properties and dynamic stability of nanoscale structures, offering new perspectives on biomimetic material design and nanotechnology applications.

In the realm of advanced composites, Bouhadra et al. [9] improved higher-order shear deformation theory (HSDT) to account for thickness stretching in composite plates. Their work significantly enhances the accuracy of structural analysis, especially for applications requiring high precision, such as aerospace and automotive engineering. Similarly, Boulefrakh et al. [10] focused on the viscoelastic behavior of thick functionally graded (FG) plates, incorporating the effects of the visco-Pasternak foundation. Their findings provide valuable insights into vibration and bending properties, critical for optimizing materials used in high-performance structural systems. Carrera and colleagues [11, 12] extended the analysis to electro-mechanical behaviors in composite and multilayered structures, using advanced shell elements with node-dependent kinematics and variable ESL/LW kinematic formulations. These methodologies allow for a comprehensive understanding of free-vibration and thermoelastic responses in laminated structures, providing a robust theoretical foundation for designing materials with tailored mechanical and thermal properties. Additionally, Cheng and Zhu [13] explored the dynamics and resonance characteristics of non-classical piezoelectric microdisks under rotary motion. Their findings contribute to the design of next-generation micro-electro-mechanical systems (MEMS) and sensors. Cinefra et al. [14] added to this body of knowledge by analyzing heat conduction and thermal stress in laminated composites using a novel MITC9 shell element, advancing computational methods for thermal analysis in layered materials.

The study of dynamic responses in piezoelectric and functionally graded materials has become increasingly important for applications involving thermal and mechanical coupling. Dai and Wang [15] made significant contributions to understanding the transient magneto-thermo-electro-elastic behavior in piezoelectric hollow cylinders. Their analysis, focused on complex loadings, provides a robust framework for examining interactions between thermal, electrical, and magnetic fields, which is critical for the design of smart materials and structures.

Building on this foundation, Heidari et al. [16] explored the nonlocal vibration characteristics of functionally graded porous cylindrical nanoshells integrated with arbitrary arrays of piezoelectric elements. This study introduced a comprehensive model for evaluating nanoscale vibrational properties, paving the way for advancements in nanotechnology and the development of multifunctional materials. Heydarpour et al. [17] extended the research to the thermoelastic analysis of rotating multilayer functionally graded graphene platelets reinforced composite (FG-GPLRC) truncated conical shells. By employing a coupled TDQM-NURBS scheme, their work provided insights into the behavior of advanced composite materials under rotational and thermal influences, making it highly relevant for aerospace and mechanical engineering applications. Similarly, Malekzadeh et al. [18] investigated the transient response of rotating laminated functionally graded cylindrical shells in thermal environments. Their research highlighted the interplay between material grading, rotational dynamics, and thermal loading, contributing to the optimization of cylindrical structures used in high-temperature and high-speed environments.

The exploration of wave propagation and stress dynamics in various thermoelastic and magnetoelastic systems continues to drive innovation in material science and fluid-structure interaction models. Mahmoud [19] investigated the effect of rotation and magnetic fields on peristaltic transport in a Jeffrey fluid through a porous medium, providing key insights into fluid dynamics under complex physical conditions. This research is particularly significant for applications involving biomedical and industrial fluid transport. Mahmoud [20-21] extended this work by analyzing Rayleigh wave propagation in granular and orthotropic media under the influence of rotation, magnetic fields, and initial stresses. These studies emphasized the critical role of non-homogeneity and gravitational effects in modifying wave behavior, offering a theoretical framework for understanding wave interactions in geophysical and structural engineering contexts.

In further developments, Mahmoud [22-23] addressed the challenges in modeling shear wave propagation in magnetoelastic half-spaces and the electrostatic potential in wave interactions within biological structures such as human long bones. These analytical solutions provide a foundation for advancing biomimetic materials and understanding the mechanical behavior of biological systems under stress. Mahmoud and Abd-Alla [24] contributed to the study of free vibrations in orthotropic hollow spheres subjected to magnetic fields, highlighting the interplay between material anisotropy and external forces. This work is critical for designing advanced materials used in aerospace, automotive, and energy industries. Expanding into thermo-viscoelastic media, Mahmoud [25] explored the combined effects of initial stress, rotation, and periodic loading in spherical cavities, presenting solutions relevant to vibration control and advanced sensing technologies. Mahmoud [26] also investigated wave propagation in cylindrical poroelastic structures, emphasizing the dynamic responses of such materials under varying boundary conditions.

Finally, Mahmoud et al. [27] introduced a novel mechanical model for analyzing mosquito bites, showcasing its potential applications in membrane filtration and water purification. This interdisciplinary approach demonstrates the versatility of advanced material analysis in addressing biological, environmental, and industrial challenges. Recent advancements in material science and structural analysis have further enriched the understanding of bending, buckling, and vibrational behaviors in composite and functionally graded materials. Meksi et al. [28] provided an analytical solution for the bending, buckling, and vibration responses of functionally graded material (FGM) sandwich plates. Their work addresses critical aspects of structural stability and mechanical performance, offering a robust theoretical model applicable to high-performance structural systems in aerospace, automotive, and marine industries. Othman and Song [29] contributed to the field by examining the rotational effects on thermal shock problems in generalized magneto-thermoelasticity. By applying multiple theories to half-space models, they highlighted the interplay between thermal, magnetic, and elastic effects, paving the way for advancements in thermal shock resistance materials and systems. Ponnusamy and Selvamani [30-31] extended these investigations to cylindrical panels, analyzing dispersion and wave propagation in magneto-thermoelastic media. Their research delves into the dynamic responses of such structures, providing insights into the dispersion characteristics under varying thermal and magnetic conditions. These studies are vital for optimizing the performance of cylindrical panels used in high-temperature environments, such as reactors and pressure vessels.

This growing body of research underscores the importance of integrating theoretical advancements with practical applications, enabling the development of materials and systems capable of withstanding extreme conditions while maintaining their mechanical integrity. The study of thermal stresses and rotational effects in piezoelectric and elastic materials continues to provide valuable insights into their dynamic and structural behaviors. Ramady et al. [32] presented a mathematical model addressing the impact of rotation on thermal stresses in piezoelectric homogeneous materials. Their research highlighted the intricate coupling between rotational dynamics and thermal effects, offering potential advancements in sensor technologies and energy harvesting systems. In a complementary study, Ramady, Mahmoud, and Atia [33] proposed a theoretical framework for analyzing periodic wave solutions in elastic bodies within a 2D-space model. This approach underscores the importance of periodicity in understanding wave propagation and stress distribution, with applications ranging from acoustic wave technology to structural health monitoring.

The contributions of Remil et al. [34] to the bending, buckling, and dynamic behavior of laminated composite plates using a simplified higher-order shear deformation theory (HSDT) have further enhanced the field. Their findings provide practical solutions for designing advanced composite materials with improved stability and durability, making them particularly relevant for aerospace and automotive industries. Saadatfar and Zarandi [35] investigated the effects of angular acceleration on exponentially graded piezoelectric rotating annular plates. Their study addresses the mechanical behavior of these structures under variable thickness conditions, highlighting their relevance in advanced rotational systems and precision engineering applications. Lastly, Tiwari and Kumar [36] explored the non-local effects on the quality factor of micro-mechanical resonators under a three-phase-lag thermoelasticity model with memory-dependent derivatives. Their findings contribute to the development of high-performance micro-resonators, which are critical components in micro-electro-mechanical systems (MEMS) and other advanced technologies.Analyzing Stress dynamics in piezoelectric thermoelastic media within inviscid fluids under magnetic and rotational influences explores the complex interactions and stress behaviors in advanced membrane technologies. These insights are crucial for enhancing applications such as antibacterial and antifouling membranes, which combat biofouling and improve water treatment efficiency. The findings will significantly impact membrane filtration, surface modification, and fouling reduction. Moreover, the study supports the development of new membrane materials and technologies, including those used for desalination, electrodialysis, and the removal of mercury and micropollutants. By advancing the understanding of stress dynamics in these materials, the research promotes sustainability and efficiency in wastewater treatment, groundwater purification, and overall water treatment processes, fostering innovation in membrane fabrication and bioreactor systems.

This study is related to wave propagation under the influence of rotation in a transversely isotropic magneto-piezoelectric thermoelastic solid bar engrossed in an inviscid fluid based on the Bessel function. The movement of potential functions, magnetic fields, and electric field vectors is implemented to uncouple the motion systems. Then, the numerical calculations are carried out and plotted as the dispersion curvatures along with their suitable properties. The findings can be applied to improve vibration analysis and control in various mechanical systems, particularly where piezoelectric and magneto-elastic effects are significant. This has implications for machinery, aerospace engineering, and even seismic research. Given the study's focus on wave propagation, it has potential applications in the field of acoustic wave technology, including in the design of more efficient sound-emitting or sound-capturing devices. The paper's research into the interaction between solid bars and surrounding fluid has implications for any system where fluid-structure interaction is critical, such as in underwater acoustics or in the design of marine vehicles, Ref. Tiwari et al.[36]. Building on the findings of this paper, future research could focus on several key areas to further advance the understanding and application of wave propagation in magneto-piezoelectric thermoelastic materials. The future research directions for this paper can significantly enhance the understanding and application of wave propagation in magneto-piezoelectric thermoelastic materials, leading to advancements in various technological and industrial fields as: Advanced Material Studies, investigating a broader range of materials, especially those with different piezoelectric and magneto-elastic properties, to understand how various material characteristics influence wave propagation in rotating environments

. Complex Fluid Dynamics, exploring more complex fluid dynamics scenarios, including turbulent or viscous fluids, to see how these conditions affect the interaction with the solid bar and wave propagation characteristics. Wave propagation in advanced composite structures, such as functionally graded polymer nanoplates, has been a pivotal area of research due to its applications in engineering and materials science. Kadiri et al. [37] explored the dynamic behavior of FG polymer composite nanoplates embedded in elastic mediums, providing insights into their stability and performance. Bourada et al. [38] introduced a refined plate theory for hybrid symmetric S-FGM plates, offering enhanced tools for stability analysis under various conditions. Similarly, Mazari et al. [39] investigated the bending response of FG thick plates with in-plane stiffness variations, contributing to the understanding of deformation behavior. The dynamic and bending analysis of carbon nanotube-reinforced plates, as studied by Bakhadda et al. [40], highlights the role of elastic foundations in optimizing plate performance. Additionally, Sadoun et al. [41] developed a quasi-3D sinusoidal shear deformation theory for thick orthotropic plates, advancing vibration analysis methods. Research on thermal environments by Taleb et al. [42] and nonlocal strain gradient elasticity by Houari et al. [43] further emphasizes the versatility and adaptability of these materials. Novel quasi-3D deformation theories for functionally graded plates, as proposed by Hebbar et al. [44] and Younsi et al. [45], underscore the ongoing evolution of analytical models to address the complexities of modern composite materials.

The fundamental equations of isotropic transversely linear magneto-elastic substantial, with strain e_k, stresses τ_j, electric movements D_{ij}, magnetic induction B_j, electric field E_k, and magnetic field H_k are measured in the Buchannan lines.

$$\tau_j = c_{jk}e_k - e_{ik}E_k - q_{kj}H_k - \beta_{ij}T\delta_{ij}. \quad (1)$$

$$D_j = e_{jk}e_k + \varepsilon_{jk}E_k + m_{jk}H_k \quad (2)$$

$$B_j = q_{jk}e_k + m_{jk}E_k + \mu_{jk}H_k \quad (3)$$

Where C_{jk}, e_{jk} and μ_{jk} is the elastic constants, piezoelectric constants, and magnetic permeability coefficients, respectively. q_{kj}, ε_{jk} and m_{jk} is the piezoelectric constants, piezo-magnetic constants, dielectric constants, and magnetoelectric material coefficients. The strain e_{ij} is linked to the displacements corresponding to the cylindrical coordinates (r, θ, z), which are given by

$$e_{rr} = \frac{\partial u}{\partial r}, \qquad e_{\theta\theta} = \frac{1}{r}\mathrm{u}, \quad (4)$$

where u is the mechanical displacement linked to the coordinate cylindrical direction r. The association between the electric potential ψ and electric field vector E_i is provided as:

$$E_r = -\frac{\partial \psi}{\partial r}\partial r, \quad (5)$$

the magnetic field H_i is linked to the magnetic potential ϕ as:

$$H_r = -\frac{\partial \phi}{\partial r}, \quad (6)$$

The basic governing equations of motion, electrostatic displacement D_j, and magnetic induction B_j in cylindrical-based coordinates (r, θ, z), under the influence of rotation, given as:

$$\frac{\partial \tau_{rr}}{\partial r} + \frac{1}{r}(\tau_{rr} - \tau_{\theta\theta}) = \rho\frac{\partial^2 u}{\partial t^2} - \rho\Omega\,\mathrm{u}, \quad (7a)$$

$$\frac{\partial D_r}{\partial r} + \frac{D_r}{r} = 0, \quad (7b)$$

$$\frac{\partial B_r}{\partial r} + \frac{B_r}{r} = 0, \quad (7c)$$

where $\overleftarrow{\boldsymbol{\Omega}} = (0,0,\Omega)$, $\left(\overleftarrow{\boldsymbol{\Omega}} \times \overleftarrow{\boldsymbol{\Omega}} \times \overleftarrow{\boldsymbol{U}}\right)_r = -\Omega\, u$ is the centripetal acceleration components in the radial-based direction $(\overleftarrow{\boldsymbol{r}})$ with the variation of time, and $\overleftarrow{\boldsymbol{U}} = (\mathrm{u}, 0, 0)$, and the relation of stress-strain based on the transversely isotropic source is provided as:

$$\tau_{rr} = c_{11}e_{rr} + c_{12}e_{\theta\theta} - \beta_1 T, \quad (8a)$$

$$\tau_{\theta\theta} = c_{11}e_{rr} + c_{12}e_{\theta\theta} - \beta_1 T, \quad (8b)$$

Magnetic induction and electric displacement are associated in the form of strain; magnetic and electric fields are

$$D_r = e_{15}e_{rz} + \varepsilon_{11}E_r + m_{11}H_r\,, \quad (9)$$

$$B_r = q_{15}e_{rz} + m_{11}E_r + \mu_{11}H_r\,, \quad (10)$$

Using Eqs. (4 to 6) and Eqs. (8 to 10) into Eq. (7), the following system is obtained in the form of displacements, magnetic and electric potential as:

$$c_{11}\frac{\partial^2 u}{\partial r^2}-(c_{12}+c_{11})\frac{1}{r^2}\mathrm{u}+\frac{1}{r}c_{11}\frac{\partial u}{\partial r}-\beta_1\frac{\partial T}{\partial r}=\rho\frac{\partial^2 \mathrm{u}}{\partial t^2}\;-\rho\Omega\,\mathrm{u} \tag{11.a}$$

$$\varepsilon_{11}\frac{\partial^2\psi}{\partial r^2}-m_{11}\frac{\partial^2\phi}{\partial r^2}-\frac{1}{r}\left(\varepsilon_{11}\frac{\partial\psi}{\partial r}+m_{11}\frac{\partial\phi}{\partial r}\right)=0 \tag{11.b}$$

$$-m_{11}\frac{\partial^2\psi}{\partial r^2}-\mu_{11}\frac{\partial^2\phi}{\partial r^2}-\frac{1}{r}\left(m_{11}\frac{\partial\psi}{\partial r}+\mu_{11}\frac{\partial\phi}{\partial r}\right)=0 \tag{11.c}$$

$$k_1\left(\frac{1}{r}\frac{\partial T}{\partial r}+\frac{\partial^2 T}{\partial r^2}\right)=\rho\,C_v\left(\frac{\partial T}{\partial t}+\tau_0\frac{\partial^2 T}{\partial t^2}\right)+T_0\,\beta_1\left(\frac{\partial^2 u}{\partial t\partial r}+\frac{1}{r}\frac{\partial \mathrm{u}}{\partial t}\right) \tag{11.d}$$

The dimensionless measures are introduced as:

$$x=\frac{r}{a},\qquad \Omega=\rho\Omega^2a^2/c_{11},\quad \bar{c}_{ij}=\frac{c_{ij}}{c_{11}},\qquad \bar{e}_{ij}=\frac{e_{ij}}{e_{31}},$$
$$\bar{q}_{ij}=\frac{q_{ij}}{q_{31}},\qquad \bar{m}_{ij}\frac{m_{ij}c_{11}}{e_{31}q_{31}},\qquad \bar{\varepsilon}_{ij}=\frac{\varepsilon_{ij}c_{11}}{e_{31}^2},\quad \bar{\mu}_{ij}=\frac{\mu_{ij}c_{11}}{q_{31}^2},$$
$$c_1^2=\frac{c_{11}}{\rho},\qquad \bar{\beta}_1=\frac{\beta_1 a^2}{c_{11}},\qquad \bar{\rho}=\frac{\rho C_v}{k_1}, \tag{12}$$

3. Solution method based on solid medium

The odd and even orders are obtained through Eqs. (11a) to (11d) are coupled with three displacements, electric components, and magnetic potentials regarding the precise coordinate variable. Using the above Eqs. (11), the updated form is given as:

$$\begin{aligned} u(r,t)&=\frac{\partial U(r)}{\partial r}e^{i\omega t},\\ \psi(r,t)&=\frac{i}{a}\Psi(r)\,e^{i\omega t},\\ \phi(r,t)&=\frac{i}{a}\,\Phi(r)\,e^{i\omega t},\\ T(r,t)&=\;\Theta(r)\,e^{i\omega t}. \end{aligned} \tag{13}$$

Where $i=\sqrt{-1},\omega$ is the angular frequency, $U(r),\Psi(r),\ \Phi(r)\ and\ \Theta(r)$ are the potential displacements for symmetric systems of vibrations, a shows the geometrical based cylindrical parameter bar.

The updated form of the Eqs. (12 to 13) into Eqs. (11a to 11d) is given as:

$$\begin{aligned} &\bar{c}_{11}\left(\nabla^2+(\Omega^2-\zeta^2)\right)U(r)-\bar{\beta}\Theta(r)=0,\\ &\bar{\varepsilon}_{11}\nabla^2\Psi(r)+\bar{m}_{11}\nabla^2\Phi(r)=0,\\ &\bar{m}_{11}\nabla^2\Psi(r)+\bar{\mu}_{11}\nabla^2\Phi(r)=0,\\ &(\nabla^2-\omega(i-\tau_0\omega)\bar{\rho}r)\Theta(r)-i\bar{\beta}\nabla^2U(r)=0. \end{aligned} \tag{14a-d}$$

Where

$$\nabla^2=\frac{\partial^2}{\partial x^2}+\frac{1}{x}\frac{\partial}{\partial x},\quad \zeta^2=\frac{1}{x}\,,\quad \bar{\beta}=\frac{\beta_1 T_0\omega}{k_1}.$$

The solutions of Eqs. (14a)– (14d) given by

$$U(r)=C_5+C_6\,J_0(\alpha_1 r)+C_7\,J_0(\alpha_2 r)+C_8\,K_0(\alpha_3 r)+C_9\,K_0(\alpha_4 r) \tag{15d}$$

$$\Psi(r)=C_1+C_2\ln(r) \tag{15a}$$

$$\Phi(r)=C_3+C_4\ln(r) \tag{15b}$$

$$\Theta(r) = B_1\big(C_8\,K_0(\alpha_5 r) + C_6\,J_0(\alpha_7 r)\big) + B_2\,\big(C_9\,K_0(\alpha_6 r) + C_7\,J_0(\alpha_8 r)\big) \qquad (15c)$$

Moreover, thus

$$\mathrm{u}(r,t) = -(C_6\,J_1(\alpha_1 r)\alpha_1 + C_7\,J_1(\alpha_2 r)\alpha_2 + C_8\,K_1(\alpha_3 r)\alpha_3 + C_9\,K_0(\alpha_4 r)\alpha_4)\,e^{i\,\omega\,t} \qquad (16a)$$

$$T(r,t) = \big(B_1\big(C_6\,J_0(\alpha_7 r) + C_8\,K_0(\alpha_5 r)\big) + B_2\big(C_7\,J_0(\alpha_8 r) + C_9\,K_0(\alpha_6 r)\big)e^{i\,\omega\,t} \qquad (16b)$$

$$\psi(r,t) = \frac{i}{a}(C_1 + C_2\ln(r))\,e^{i\,\omega\,t} \qquad (16c)$$

$$\phi(r,t) = \frac{i}{a}(C_3 + C_4\ln(r))\,e^{i\,\omega\,t} \qquad (16d)$$

The stress we rewrite as

$$\tau_{rr} = -c_{11}\left(C_6\left(J_0(\alpha_1 r) - \frac{J_1(\alpha_1 r)}{\alpha_1 r}\right)\alpha_1^2 + C_7\left(J_0(\alpha_2 r) - \frac{J_1(\alpha_2 r)}{\alpha_2 r}\right)\alpha_2^2 + C_8\left(K_0(\alpha_3 r) - \frac{K_1(\alpha_3 r)}{\alpha_3 r}\right)\alpha_3^2 + C_9\left(K_0(\alpha_4 r) - \frac{K_1(\alpha_4 r)}{\alpha_4 r}\right)\alpha_4^2\right)e^{i\omega t} - \frac{1}{r}\big(c_{12}(C_6\,J_1(\alpha_1 r)\alpha_1 + C_7\,J_1(\alpha_2 r)\alpha_2 + C_8\,K_1(\alpha_3 r)\alpha_3 + C_9\,K_1(\alpha_4 r)\alpha_4)\,e^{i\,\omega\,t}\big) - \beta_1\left(B_1\big(C_8\,K_0(\alpha_5 r) + C_6\,J_0(\alpha_7 r)\big) + B_2\,\big(C_9\,K_0(\alpha_6 r) + C_7\,J_0(\alpha_8 r)\big)\right)e^{i\,\omega\,t} \qquad (17a)$$

$$\tau_{\theta\theta} = -c_{12}\left(C_6\left(J_0(\alpha_1 r) - \frac{J_1(\alpha_1 r)}{\alpha_1 r}\right)\alpha_1^2 + C_7\left(J_0(\alpha_2 r) - \frac{J_1(\alpha_2 r)}{\alpha_2 r}\right)\alpha_2^2 - C_8\left(K_0(\alpha_3 r) + \frac{K_1(\alpha_3 r)}{\alpha_3 r}\right)\alpha_3^2 - C_9\left(K_0(\alpha_4 r) + \frac{K_1(\alpha_4 r)}{\alpha_4 r}\right)\alpha_4^2\right)e^{i\omega t} - \frac{1}{r}\big(c_{11}(C_6\,J_1(\alpha_1 r)\alpha_1 + C_7\,J_1(\alpha_2 r)\alpha_2 + C_8\,K_1(\alpha_3 r)\alpha_3 + C_9\,K_1(\alpha_4 r)\alpha_4)\,e^{i\,\omega\,t}\big) - \beta_1\left(B_1\big(C_8\,K_0(\alpha_5 r) + C_6\,J_0(\alpha_7 r)\big) + B_2\,\big(C_9\,K_0(\alpha_6 r) + C_7\,J_0(\alpha_8 r)\big)\right)e^{i\,\omega\,t} \qquad (17b)$$

$$D_r = -\frac{i}{r\,a}(C_2\,\varepsilon_{11} + C_4 m_{11})\;e^{i\omega t} \qquad (18)$$

$$B_r = -\frac{i}{r\,a}(C_2\,m_{11} + C_4\mu_{11})\;e^{i\omega t} \qquad (19)$$

Where $B_j,\ j = 1,2,3$ and $\alpha_i,\ i = 1,2,3,\ldots,8$ are given as

$$B_1 = \left((\tau_0\omega - i)\big(\sqrt{A_3 - A_4} + A_5\big)\right),\ B_2 = -\left((\tau_0\omega - i)\big(\sqrt{A_3 - A_4} + A_5\big)\right) = -B_1,$$

$$B_3 = \frac{1}{2}\frac{\omega}{k_1\beta_1(-\tau_0\omega + i)},$$

and

$$\alpha_1 = \frac{1}{2}\sqrt{-\frac{2\big(A_2 - \sqrt{A_1}\big)\omega}{c_{11}k_1 + \Omega}}, \qquad \alpha_2 = \frac{1}{2}\sqrt{-\frac{2\big(A_2 + \sqrt{A_1}\big)\omega}{c_{11}k_1 + \Omega}},$$

$$\alpha_3 = -\frac{1}{2}i\sqrt{-\frac{2\big(A_2 - \sqrt{A_1}\big)\omega}{c_{11}k_1 - \Omega}}, \qquad \alpha_4 = -\frac{1}{2}i\sqrt{-\frac{2\big(A_2 + \sqrt{A_1}\big)\omega}{c_{11}k_1 - \Omega}},$$

$$\alpha_5 = -i\frac{1}{2}\sqrt{-\frac{2\left(-\sqrt{A_3-A_4}+A_6\right)\omega}{c_{11}k_1}}, \alpha_6 = -i\frac{1}{2}\sqrt{-\frac{2\left(\sqrt{A_3-A_4}+A_6\right)\omega}{c_{11}k_1}},$$

$$\alpha_7 = \frac{1}{2}\sqrt{-\frac{2\left(-\sqrt{A_3-A_4}+A_6\right)\omega}{c_{11}k_1}}, \qquad \alpha_8 = \frac{1}{2}\sqrt{-\frac{2\left(\sqrt{A_3-A_4}+A_6\right)\omega}{c_{11}k_1}},$$

and

$$A_1 = C_v^2\omega^2\rho^2\tau_0^2\, c_{11}^2 + 2C_vT_0\omega^2\rho\tau_0^2\beta_1^2\, c_{11} + T_0^2\omega^2\tau_0^2\beta_1^4 - 2iC_v^2\omega\rho^2\tau_0c_{11}^2 - 4iC_vT_0\omega\rho\tau_0\beta_1^2c_{11} - 2iT_0^2\omega\tau_0\beta_1^4 - 2C_v\omega^2\rho^2\tau_0c_{11}k_1 + 2T_0\omega^2\rho\tau_0\beta_1^2k_1 + 2iC_v\omega\rho^2c_{11}k_1 - 2iT_0\omega\rho\beta_1^2k_1 - C_v^2\rho^2c_{11}^2 - 2C_vT_0\rho\beta_1^2c_{11} - T_0^2\beta_1^4 + \omega^2\rho^2k_1^2,$$

$$A_2 = -\, C_v\omega\rho\tau_0c_{11} - T_0\omega\tau_0\beta_1^2 + i\, C_v\rho c_{11} + i\, T_0\beta_1^2 - \omega\rho k_1,$$

$$A_3 = ((C_v\tau_0c_{11} - k_1)^2\rho^2 + 2T_0\tau_0\beta_1^2(C_v\tau_0c_{11} + k_1)\rho + T_0^2\tau_0^2\beta_1^4)\omega^2 + \left((-i\, c_{11}(C_v\tau_0c_{11} - k_1))C_v\rho^2 - 4i\, T_0\left(C_v\tau_0c_{11} + \frac{1}{2}k_1\right)\beta_1^2\rho - 2i\, T_0^2\tau_0\beta_1^4\right)\omega,$$

$$A_4 = (C_v\rho c_{11} + T_0\beta_1^2)^2,$$

$$A_5 = ((C_v\rho c_{11} + T_0\beta_1^2)\tau_0^2 - k_1\rho\tau_0)\omega^2 - ((2iC_v\rho c_{11} + 2iT_0\beta_1^2)\tau_0 - ik_1\rho)\omega - C_v\rho c_{11} - T_0\beta_1^2,$$

$$A_6 = (-\tau_0\omega + i)C_v\rho c_{11} - \omega\rho k_1 + T_0(-\tau_0\omega + i)\beta_1^2,$$

4. Motion equations of the fluid

In the context of acoustic pressure and radial displacement equations of motion expressed in cylindrical polar coordinates, the representation of the inviscid fluid adheres to the formulation proposed by Berliner and Solecki [20]. Their work provides a fundamental framework for understanding the behavior of such fluids under acoustic influence, particularly in the realm of cylindrical coordinate systems.

$$p^f = -A^f\left(u_{,r}^f + \frac{1}{r}u^f\right) \tag{20}$$

$$\bar{c}_f^2 u_{,tt}^f = \Delta_{,r} \tag{21}$$

where A^f indicates the modulus of the adiabatic bulk, ρ^f is the density,

$$c_f = \sqrt{A^f/\rho^f}$$

shows the velocity of the acoustic phase in the fluid

$$\Delta = \left(u_{,r}^f + \frac{1}{r}u^f\right) \tag{22}$$

Substituting

$$u^f = \psi_{,r}^f \tag{23}$$

Moreover, the Eq. (21) takes the form

$$\psi^f(r,t) = \Psi^f(r)\, e^{i\omega T_a}, \tag{24}$$

The fluid that shows the oscillatory wave propagating is represented as

$$\Psi^f = A_1 + A_2 r^2 + A_3 r^{\frac{\bar{c}_f^2}{\bar{c}_f^2-1}} \tag{25}$$

$$u^f(r,t) = \left(2A_2 r + \frac{A_3 \bar{c}_f^2}{\left(\bar{c}_f^2 - 1\right) r} r^{\frac{\bar{c}_f^2}{\bar{c}_f^2-1}} \right) e^{i\omega T_a}, \tag{26}$$

where A_i , $i = 1,2,3$, are constants, which can be determined from BCs and

$\bar{c}_f = \frac{c_f}{c_{11}}$.

Using Eq. (24) in Eq. (20) together with Eq.(23), the acoustic pressure based on the fluid can be stated as

$$p^f = -4A^f \left(\frac{A_2 \left(\bar{c}_f - 1\right)^2 \left(\bar{c}_f + 1\right)^2 r^2 + \frac{1}{4} A_3 \bar{c}_f^4 r^{\frac{\bar{c}_f^2}{\bar{c}_f^2-1}}}{\left(\bar{c}_f^2 - 1\right)^2 r^2} \right), \tag{27}$$

5. Frequency equations and boundary conditions

This study addresses the free vibration of a rotating, transversely isotropic, piezoelectric solid bar with a circular cross-section, submerged in a fluid. It focuses on the interface dynamics between the solid and fluid, where the bar's normal stress equals the fluid's exerted negative pressure. Additionally, the displacement along the bar's lateral surface in the normal direction matches the fluid's displacement in the same direction, reflecting the continuity of stresses and displacements at the solid-fluid boundaries. Given that the inviscid fluid cannot support shear stress, the shear stress at the fluid boundary is null. In solid-fluid interaction problems, continuity conditions necessitate equivalency in displacement components, surface stress components, and electric potential at the interface. These conditions are integral to defining the boundary conditions for the system.

$$\tau_{rr} + p^f = 0, \quad u - u^f = 0, \quad at \;\; r = a, \tag{28}$$

$$D_r = 0 \qquad and \qquad B_r = 0 \tag{29}$$

6. Results and discussion

In this work, wave propagation of transversely isotropic magneto-electro-elastic solid bar, under the influence of rotation, immersed in a fluid is considered. In the context of solid-fluid interface problems, the principle of continuity for displacements and stresses at the boundaries of the fluid and solid is critical. This continuity ensures that the negative pressure values exerted by the fluid are equivalent to the normal stress experienced by the bar. Similarly, the displacement of the fluid in a given direction matches the displacement of the corresponding component in the normal direction of the cylinder's lateral surface. Furthermore, due to the nature of the inviscid fluid, which is incapable of sustaining shear-based stress, the shear stress at the outer fluid boundary is effectively zero. This aspect is fundamental in understanding the interactions and stress distributions in such solid-fluid systems. The material properties of the electromagnetic material are based on Refs.[29-31]

$$c_{11} = 2.11 \times 10^{10}, \quad c_{12} = 0.94 \times 10^{10}, \quad c_{13} = 1.02 \times 10^{10}, \quad m_{11} = 0.0074 \times 10^{-9},$$
$$e_{15} = 0, \quad q_{15} = 200, \quad q_{31} = 265, \quad \varepsilon_{11} = 0.4 \times 10^{-11}, \quad \mu_{11} = -200 \times 10^{-6},$$
$$p_f = 1000, \quad c_f = 1500, \quad \rho = 5.3 \times 10^3, \quad C_v = 420, \quad T_0 = 298, \quad \beta_1 = 1.52 \times 10^6,$$
$$\beta_3 = 1.53 \times 10^6, \quad p_3 = -452 \times 10^{-6}, \quad e_{31} = -2.5 \times 10^{10}, \quad k_1 = 1.5, \quad k_3 = 1.5.$$

And the units of the onstants c_{ij} in N/m^2 , e_{ij} in C/m^2 , ε_{ij} in F/m, ρ in kg/m^3 .

Figures (1-4) Show the dispersion curves under the influence of the rotation for the results of the real part of longitudinal modes. In Figures 1 and 2, the non-dimensional variation frequency under the influence of the rotation, concerning the wavenumber has been shown for the real longitudinal vibration parts for the bar engrossed in fluid using the ratio of velocity v_R = 0.6, and 1.0. In Figure 1, it is noticed that the non-dimensional electromagnetic frequency bar under the influence of the rotation indicates the linear variation concerning the wavenumber for the velocity ratio v_R = 0.6. Figure 2 shows the oscillating nature in the start domain from the linear frequency conduct because of the influence of the rotation and the damping impacts of nearby fluid medium, and the enhancement in velocity profile is more projected in the lower wavenumber. In Figures 1 and 2, it is also noticed that in the presence of rotation, the real frequency mode is a smaller amount with ignored rotation because of the damping based on the fluid medium. The dispersal of attenuation coefficient concerning the wavenumber, under the influence of the rotation, is studied for the immersed types and a free elastic magnetoelectric solid bar in Figures 3 and 4. The displacement amplitude of the lessening coefficient enhances monotonically to get the maximum results in the start domain, and the dotted line reduces to get the form of asymptotically linear in the enduring wavenumber range in Figure 3. The disparity in weakening coefficient under the influence of the rotation for real portions of longitudinal methods is wavering in the maximum wavenumber range, as described in Figure 4. In Figures 3 and 4, it is indicated that the attenuation states under the influence of the rotation display high oscillating form in the velocity term $v_R = 1.0$ than $v_R = 0.6$ because of the combined impacts of surrounding fluid and magnetic fields. The crossover terms in the vibrational forms designate the energy transfer between the fluid and solid standard under the influence of the rotation.

According to the results of the numerical calculations above, you will notice that all physical quantities under the influence of the rotation overlap with the boundary conditions. Lastly, these outcomes also deliver engineers and designers a valuable standard for modeling and design solutions. It is observed that the analytical form of the solutions based on analysis of normal mode to medium heat elastic piezoelectric, under the influence of rotation, have been developed and used, and all the functions are continuous. All physical measures values meet to a very small value with the enhancing distance. The study's insights are highly valuable for the development of sophisticated sensors and transducers, which are crucial in numerous industrial and technological applications. These devices, which leverage the interaction between magnetic and electric fields, can be optimized for enhanced sensitivity and performance in rotating environments. The research offers significant contributions to the field of composite materials, especially those used in rotating systems. Understanding the wave propagation in these materials under the influence of rotation and magnetic fields can lead to the creation of more efficient and robust composite structures.

The research on analyzing stress dynamics in piezoelectric thermoelastic media within inviscid fluids under magnetic and rotational influences has vital applications in several areas within membrane technology. It can significantly enhance antibacterial and antifouling properties by improving material design to resist microbial growth and fouling. In membrane filtration, fouling, and surface modification, the insights gained can lead to the development of more efficient and durable membranes. This research also contributes to advancements in the removal of mercury and micropollutants from water, optimizing processes like electrodialysis and desalination [33-36]. Additionally, it supports the innovation of new membrane materials and improves the performance of membrane bioreactors, distillation, and wastewater treatment technologies, ultimately promoting sustainability in water treatment applications.

The advanced methodologies applied in this study provide a robust foundation for analyzing the complex interplay between magnetic and rotational influences on piezoelectric thermoelastic media. Fourn et al. [46] introduced a novel refined plate theory that significantly enhances wave propagation analysis in functionally graded material plates, offering a deeper understanding of structural dynamics. Additionally, Bouhadra et al. [47] highlighted the importance of thickness stretching effects in advanced composite plates, which directly correlates with the accuracy of stress and deformation predictions. The inclusion of shear deformation theories, as presented by Mokhtar et al. [48], and the buckling analysis of composite beams by Kaci et al. [49], further solidifies the theoretical framework underpinning this study. These contributions, combined with the refined thermoelastic models proposed by Attia et al. [50] for plates resting on variable elastic foundations, underscore the study's alignment with cutting-edge advancements. Moreover, the application of hyperbolic shear deformation theory for vibration and bending analysis, as detailed by Belabed et al. [51], and higher-order theories for isotropic and multilayered plates from Zine et al. [52], emphasizes the versatility of the approaches utilized. The findings resonate strongly with the novel plate theories tailored for free vibration and bending analysis of advanced composite structures, as elaborated by Abualnour et al. [53] and Benchohra et al. [54]. These integrated methodologies provide a comprehensive framework for addressing the intricate dynamics of piezoelectric thermoelastic systems, paving the way for future explorations in both theoretical and applied mechanics.

7. Conclusion

In conclusion, the research on "Analyzing Stress Dynamics in Piezoelectric Thermoelastic Media within Inviscid Fluids under Magnetic and Rotational Influences" reveals significant advancements across multiple domains. Enhanced antibacterial and antifouling properties, improved membrane filtration, and reduced fouling demonstrate the broad applicability of these findings. Furthermore, the insights into membrane surface modification, new materials, and technologies such as desalination, electrodialysis, and micropollutant removal underscore the potential for improved water treatment processes. The study also highlights the importance of sustainability and innovation in wastewater treatment, groundwater purification, and membrane bioreactors, paving the way for future developments in membrane technology. In this study, the propagation of waves in a magneto-piezoelectric thermoelastic solid bar immersed in fluid, influenced by rotation, is explored within the context of a foundational linear magneto-electro-elastic theory, adhering to specified boundary conditions. The frequency characteristics of the solid bar are determined with respect to various velocity terms. The present research computes and graphically represents the phase velocity, dimensionless frequency, and attenuation coefficient, incorporating and excluding rotational effects, through the utilization of real longitudinal vibration modes. The numerical results distinctly illustrate that the enhancement of velocity terms and the introduction of a fluid source significantly affect all frequency modes, attenuation, and phase velocity. Furthermore, the derived outcomes are deemed valuable for the analysis and design of rotating magnetoelectric sensors and transducers, particularly those employing rotating composite materials. In summary, this paper's findings have broad applications across various engineering and technological domains, particularly in the enhancement of sensor technology, composite material design, vibration control, acoustic wave technology, and fluid-structure interaction systems.

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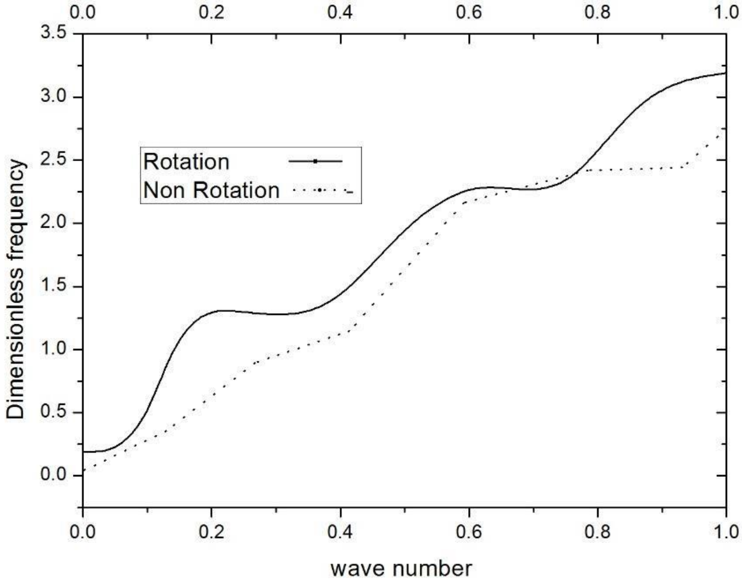

Figure 1 Dispersion curves for dimensionless frequency vs wave number with dissimilar cases (…Ω =0.0, — Ω =0.5) with the velocity ratio = 0.5

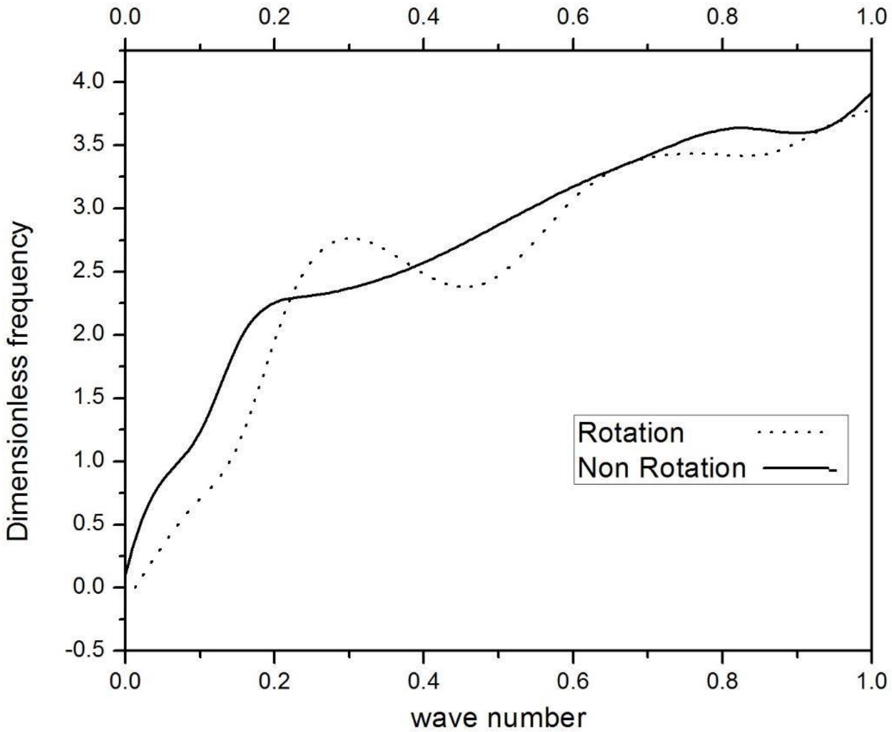

Figure 2 Dispersion curves for dimensionless frequency, vs wave number with dissimilar cases (…Ω =0.0, — Ω =0.5) with the velocity ratio = 1.0

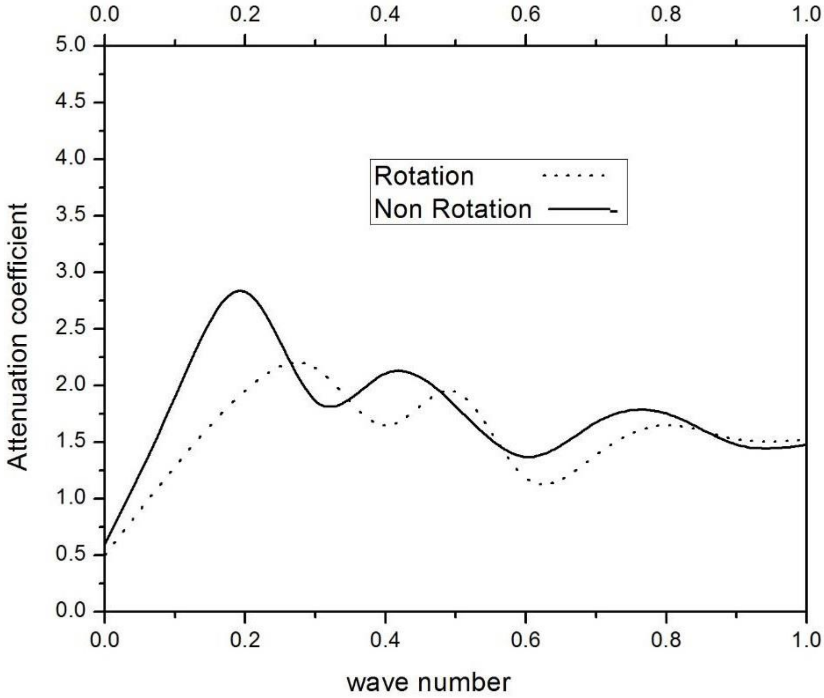

Figure 3 Dispersion curves based on the attenuation coefficient, vs wave number with dissimilar cases (…Ω =0.0, — Ω =0.5) with the velocity ratio = 0.5

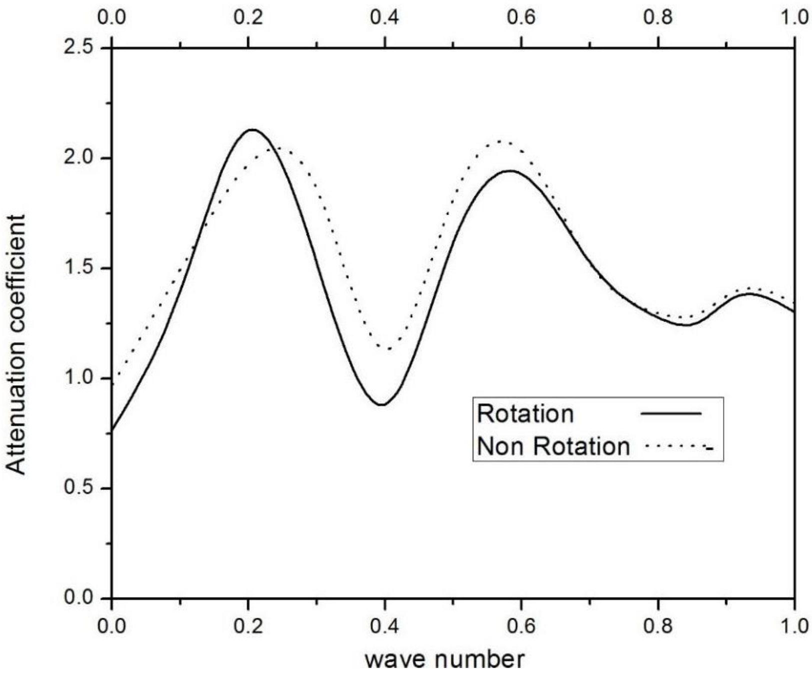

Figure 4 Dispersion curves for attenuation coefficient, vs wave number with dissimilar cases (…Ω =0.0, — Ω =0.5) with the velocity ratio = 1.0

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