Deformation Dynamics of Fiber-Reinforced Anisotropic Elastic Media Under Combined Magnetic, Rotational, and Gravitational Effects

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Abstract.

This research article investigates the impact of rotation and gravity on the deformation of a fiber-reinforced elastic medium subjected to a magnetic field, considering the presence of initial hydrostatic stress. The study delves into the theory of generalized surface waves and applies it to analyze specific cases. An analytical solution to the Equations of motion for the fiber-reinforced elastic medium, considering specific boundary conditions, is derived. The dispersion curves for displacement components and normal and shear stress components are examined, taking into account the influence of rotation, initial stress, magnetic field, and gravity field. The numerical results are presented and visually depicted for each variable of interest. Lastly, comprehensive comparisons are conducted between scenarios with and without initial hydrostatic stress, rotation, gravity field, and magnetic field, facilitating a thorough understanding of their combined effects.

Keywords: Rotation, Animal prosthetics, Fiber-reinforced, Magnetic field, Gravity, Anisotropic, Initial stress.

1. Introduction

The study of the effects of rotation, gravity, magnetic fields, and initial hydrostatic stress on the deformation of fiber-reinforced elastic materials has significant applications in veterinary science and agricultural practices. Such research provides valuable insights for designing advanced bio-materials, including prosthetics for animals, and durable structures such as shelters and farm facilities capable of withstanding various stressors in demanding environments. Furthermore, the findings contribute to the development of stress-resistant feeding and transport systems, ensuring the safety and efficiency of animal management. Additionally, the use of reinforced elastic materials in veterinary building materials and robust storage solutions for feed and animal products offers practical solutions for improving sustainability and resilience in the agricultural and veterinary sectors. These interdisciplinary applications underscore the importance of bridging material science with real-world needs in the fields of veterinary and agricultural engineering.

The fiber-Reinforced Polymer materials are composite materials comprising fibers with a polymer matrix reinforced in the shape of the fabric, strands, mat, or any other fiber form. Fiber-reinforced composites are utilized in many structures due to their few weight and top strength. Materials, for example, resins reinforced by strong stratified fibers exhibit highly anisotropic elastic or mechanical behavior in the sense that their elastic constants for extension in the fiber direction are repeatedly of the order of fifty or greater than their elastic constants in transverse extension or in shear. [1]. Fiber-Reinforced Polymer (FRP) composites consist of solid fibers embedded in a resin matrix. The matrix protects the fibers and facilitates stress transfer between them through shear stresses. Various reinforcing materials, such as glass, wood, carbon, and aramid fibers, as well as inorganic particulates and microspheres, offer improved properties such as strength, elasticity, heat resistance, and weight reduction. These fibers contribute strength and stiffness to the composite, carrying most of the applied loads. FRP composites are known for their high strength, nonconductive, noncorrosive, and lightweight nature. [2]. Recent focus has been on the dynamics of surface wave propagation in non-homogeneous and homogeneous elastic and thermoplastic media. This is significant for engineering and earthquake seismology due to the presence of non-homogeneities in the earth's crust, with multiple distinct layers.[3-4] Attention is drawn to understanding wave behavior in such materials for practical applications and geological studies. The unique characteristic of reinforced composites lies in the unity of their components, which act collectively as units within a single anisotropic structure while in the elastic state. The study of the interaction between magnetic fields and stress in thermos-elastic materials holds significant importance due to its wide-ranging applications in fields such as plasma physics, geophysics, nuclear engineering, and related areas. These applications encompass scenarios where extreme temperatures, temperature gradients, and internal nuclear reactors' magnetic fields influence the design considerations [5-6]. The design of reinforced concrete members takes into account various stress conditions based on fundamental mechanical principles. Fiber-reinforced composites are highly preferred in numerous applications due to their lightweight nature and exceptional strength. Hence, it is crucial to undertake a thorough investigation and establish an analytical framework that comprehensively captures the intricate interplay of initial stress, magnetic field, and periodic loading in a rotating thermo-viscoelastic medium containing a spherical cavity [7-10]. Prior research has made significant contributions to related areas. Chattopadhyay et al. [11] extensively examined the reflection phenomena of quasi-SV and quasi-P waves at the free and rigid boundaries of fiber-reinforced elastic media. Singh [12] conducted a comprehensive investigation into wave propagation characteristics within transversely isotropic fiber-reinforced elastic media exhibiting incompressible properties. Additionally, Singh [13] focused on the analysis of wave propagation in thermally conductive linear fiber-reinforced composite materials. In a recent study, Othman and Lotfy [14] investigated the influence of magnetic fields on the two-dimensional problem of fiber-reinforced thermos-elasticity under rotation, considering three theories that account for the effects of gravity.

These prior studies have provided valuable insights into relevant aspects and laid the foundation for further exploration in the present investigation. By building upon their findings, the current study aims to advance our understanding of the complex interactions within the rotating thermo-viscoelastic medium, shedding light on the influence of initial stress, magnetic field, and periodic loading. Additional research efforts have delved into various aspects of fiber-reinforced composites. Wu and Liu [15] studied ultrasonic bulk waves and laser-generated Lamb waves for measuring anisotropic elastic constants in composite plates. Rhee et al. [16] examined the group velocity variation of Lamb waves in fiber-reinforced composite plates. Fu and Zhang [17] investigated band formation in fiber-reinforced composites using continuum-mechanical modeling of kink bands. Espinosa et al. [18] introduced a model that incorporates induced delamination and contact/cohesive laws in woven fiber-reinforced composites. Weitsman and Benveniste [19] explored wave propagation in bi-directional fiber-reinforced materials. Weitsman [20] considered energy scattering and wave propagation in elastic materials reinforced by inextensible fibers. Dai and Wang [21] focused on stress wave propagation in piezo-electrically affected and thermally shocked fiber-reinforced laminated composites. Ohyoshi [22] investigated the propagation of directional fiber-reinforced composites' Rayleigh waves along obliquely cut surfaces. Rogerson [23] delved into the analysis of impact wave penetration in six-ply fiber composite laminates. Weitsrian [24] examined the reflection characteristics of harmonic waves in fiber-reinforced materials. Huang et al. [25] explored the scattering phenomenon in multilayered fibers and investigated the influence of the fiber-matrix interphase on wave propagation along and within composites, utilizing the transfer matrix approach. Fu and Zhang [26] conducted a comprehensive modeling study focusing on the formation of kink bands in fiber-reinforced composites, employing continuum mechanics. Singh and Singh [27] investigated the reflection behavior of plane waves at the free surface of an elastic half-space reinforced with fibers. Sengupta and Nath [28] explored the characteristics of surface waves in anisotropic elastic media reinforced with fibers. Samal and Chattaraj [29] discussed the propagation properties of surface waves in a fiber-reinforced anisotropic elastic layer sandwiched between a uniform liquid layer and a liquid-saturated porous half-space. Mahmoud et al. addressed the impact of initial stress on the radial vibrations in rotating orthotropic non-homogeneous spheres. This research paper aims to investigate the propagation behavior of surface waves in a fiber-reinforced anisotropic elastic medium under the influence of rotation, gravitational field, initial stress, and magnetic field.

The study utilizes harmonic vibrations to obtain displacement components, normal stresses, and shear stresses in the physical domain. The research simultaneously examines the effects of gravity, magnetic field, and fiber-reinforced material parameters on displacement components and stresses. Analytical expressions for surface waves are presented graphically for different scenarios, and numerical results are provided for the physical domain considering the presence or absence of rotation,

gravity field, initial stress, and magnetic field.

2. Formulation of the problem

Let the orthogonal Cartesian coordinate's oxyz , and the axis oy point vertically downward to the fibre-reinforced anisotropic solid elastic medium. The investigation under consideration explores the potential existence of a specific type of wave propagating in the 0x direction, characterized by its localized disturbance confined within a particular boundary. At any given moment, all particles aligned parallel to the y-axis exhibit equal displacements. Consequently, all partial derivatives with respect to z become either zero or negligible. To delve deeper into this phenomenon, let us consider the components of displacement (u,v) at any given point (x, y, z) and time t. It is postulated that the combined effects of the gravitational field and rotation give rise to an initial hydrostatic stress, stemming from a gradual process of creep wherein the shearing stress components tend to diminish or dissipate over an extended duration. The Equation governing displacement in the rotating frame incorporates two supplementary terms, which warrant further investigation. ; the $2\Omega \wedge u$ is the Coriolis acceleration. Term centripetal acceleration and the term $\overrightarrow{\Omega} \times (\overrightarrow{\Omega} \times \vec{u}) = (-\Omega^2 u, -\Omega^2 v, 0)$ due to time-varying motion only, where $\overrightarrow{\Omega} = (0,0,\Omega)$, $\vec{u} = (u, v, 0)$. Maxwell equations for the electromagnetic field under consideration.

$$curl\,\overleftarrow{h} = \vec{J}\,, \quad curl\,\overleftarrow{E} = -\mu_e \frac{\partial \overleftarrow{h}}{\partial t}, \quad div\overleftarrow{h} = 0, \; div\overleftarrow{E} = 0, \qquad (1a)$$

$$\vec{h} = curl\left(\vec{u} \times \vec{H_o}\right), \qquad \overleftarrow{H} = \overleftarrow{H_o} + \overleftarrow{h}, \quad \overleftarrow{H_o} = (0, 0, H_o). \quad (1b)$$

Maxwell's stresses $\bar{\tau}_{ij}$ take the form:

$$\bar{\tau}_{ij} = \mu_e\left[H_i h_j + H_j h_i - \left(\vec{H}.\vec{h}\right)\delta_{ij}\right], \qquad i, j = 1,2,3 \qquad (1c)$$

Hence, it becomes possible to derive the components of the vector representing Lorentz's body forces. The constitutive equations explicitly define the behavior of a linearly elastic anisotropic homogeneous medium that incorporates fiber reinforcement, taking into account a designated preferred direction referred to as $\vec{a}$. This notation is supported by prior research [1].

$$\tau_{ij} = \lambda\, e_{kk}\, \delta_{ij} + 2\,\mu_T\, e_{ij} + \alpha\left(a_k a_m e_{km}\delta_{ij} + e_{kk}\, a_i\, a_j\right) + 2(\mu_L - \mu_T)\left(a_i\, a_k\, e_{kj} + a_j\, a_k\, e_{ki}\right) + \beta\left(a_k a_m e_{km}\, a_i\, a_j\right) \quad (2)$$

Where, $e_{ij} = \frac{1}{2}\,(u_{i\,.j} + u_{j\,.i}\,)$ are components of strain, $\alpha,\ \beta, (\mu_L - \mu_T)$ are the anisotropic elastic parameters are enhanced, with μ_L , μ_T representing the specific elastic parameters under consideration.

$$\vec{a} = (a_1,\ a_2\,,\ a_3), \qquad {a_1}^2 + {a_2}^2 + {a_3}^2 = 1\,.$$

In the case where the components of $\vec{a}$ are aligned with the x-axis, denoted as (1, 0, 0), the analysis becomes simplified, as described below. Specifically, for the deformation occurring within the x-y plane under plane strain conditions, the stress components can be expressed in the following manner:

$$\tau_{xx} = (\lambda + 2\alpha + 4\mu_L - 2\mu_T + \beta + P)\frac{\partial u}{\partial x} + (\lambda + \alpha + P)\frac{\partial v}{\partial y}, \tag{3a}$$

$$\tau_{xy} = \left(\mu_L - \frac{P}{2}\right)\frac{\partial v}{\partial x} + \left(\mu_L + \frac{P}{2}\right)\frac{\partial u}{\partial y} \tag{3b}$$

$$\tau_{yx} = \left(\mu_L + \frac{P}{2}\right)\frac{\partial v}{\partial x} + \left(\mu_L - \frac{P}{2}\right)\frac{\partial u}{\partial y}, \tag{3c}$$

$$\tau_{yy} = (\lambda + \alpha + P)\frac{\partial u}{\partial x} + (\lambda + 2\mu_T + P)\frac{\partial v}{\partial y}, \tag{3d}$$

The equilibrium equation of the initial stress is in the form:

$$\frac{\partial \tau}{\partial x} = 0, \quad \frac{\partial \tau}{\partial y} + \rho g = 0. \tag{4}$$

The incorporation of initial stress P is a significant aspect in the study. The equations and constitutive relations for this medium, considering the presence of initial stress and heat sources, are derived and analyzed.

$$(\lambda + 2\alpha + 4\mu_L - 2\mu_T + \beta + P)\frac{\partial^2 u}{\partial x^2} + \left(\lambda + \alpha + \mu_L + \frac{p}{2}\right)\frac{\partial^2 v}{\partial x \partial y} + \left(\mu_L - \frac{P}{2}\right)\frac{\partial^2 u}{\partial y^2}$$

$$+\rho\, g\,\frac{\partial v}{\partial x} - \mu_e H_0^2\,\frac{\partial}{\partial x}\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) = \rho\left(\frac{\partial^2 \mathrm{u}}{\partial \mathrm{t}^2} - 2\Omega\frac{\partial \mathrm{v}}{\partial \mathrm{t}} - \Omega^2\,\mathrm{u}\right), \tag{5a}$$

$$(\lambda + 2\mu_T + P)\frac{\partial^2 v}{\partial y^2} + \left(\lambda + \alpha + \mu_L + \frac{P}{2}\right)\frac{\partial^2 u}{\partial x \partial y} + \left(\mu_L + \frac{P}{2}\right)\frac{\partial^2 v}{\partial x^2} - \rho\, g\,\frac{\partial u}{\partial x}$$

$$- \mu_e H_0^2\,\frac{\partial}{\partial y}\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}\right) = \rho\left(\frac{\partial^2 \mathrm{v}}{\partial \mathrm{t}^2} + 2\Omega\frac{\partial \mathrm{u}}{\partial \mathrm{t}} - \Omega^2\,\mathrm{v}\right), \tag{5b}$$

In the context of this study, the symbol ρ represents the density of the material medium, while the symbol g denotes the acceleration due to gravity. and $\tau_{ij}\ \forall\,(i, j = 1,2,3)$ are the normal stress components.

3. Solution of the problem

In this particular section, the application of Helmholtz's theorem [30-33] is employed, allowing the expression of the displacement vector $\vec{u}$ in the following form:

$$\vec{u} = \text{grad}\ \varphi + \text{curl}\ \vec{\psi}\,. \tag{6}$$

Here, the scalar φ and the vector $\vec{\psi}$ represent both the irrotational and rotational aspects of the displacement. It is reasonable to assume that only one component of the vector $\vec{\psi}$ is non-zero, thereby simplifying the analysis.

$$\vec{\psi} = (0, 0, \psi)\,. \tag{7}$$

From Esq. (6) and (7), one obtains:

$$u = \frac{\partial\varphi}{\partial x} + \frac{\partial\psi}{\partial y}\ ,\quad v = \frac{\partial\varphi}{\partial y} - \frac{\partial\psi}{\partial x}. \tag{8}$$

Substituting Equation (8) into equations (5a, b), one gets:

$$(\lambda + 2\alpha + 4\mu_L - 2\mu_T + \beta + P - \mu_e H_0^2)\frac{\partial^2\varphi}{\partial x^2} + \left(\lambda + \alpha + 2\mu_L - \mu_e H_0^2\right)\frac{\partial^2\varphi}{\partial y^2} - \rho\, g\,\frac{\partial\psi}{\partial x}$$
$$= \rho\,\frac{\partial^2\varphi}{\partial t^2} + 2\Omega\frac{\partial\psi}{\partial t} - \Omega^2\,\varphi, \tag{9}$$

$$\left(\alpha + 3\mu_L - 2\mu_T + \beta + \frac{P}{2}\right)\frac{\partial^2\psi}{\partial x^2} + \left(\mu_L - \frac{P}{2}\right)\frac{\partial^2\psi}{\partial y^2} + \rho\, g\,\frac{\partial\varphi}{\partial x} = \rho\,\frac{\partial^2\psi}{\partial t^2} - 2\Omega\frac{\partial\varphi}{\partial t} - \Omega^2\psi\,, \tag{10}$$

$$(\lambda + \alpha + 2\mu_L + P - \mu_e H_0^2)\frac{\partial^2\varphi}{\partial x^2} + (\lambda + 2\mu_T + P - \mu_e H_0^2)\frac{\partial^2\varphi}{\partial y^2} - \rho\, g\,\frac{\partial\psi}{\partial x}$$
$$= \rho\,\frac{\partial^2\varphi}{\partial t^2} + 2\Omega\frac{\partial\psi}{\partial t} - \Omega^2\varphi\,, \tag{11}$$

$$-\left(\frac{P}{2} + \mu_L\right)\frac{\partial^2\psi}{\partial x^2} + \left(\alpha + \mu_L - 2\mu_T - \frac{P}{2}\right)\frac{\partial^2\psi}{\partial y^2} - \rho\, g\,\frac{\partial\varphi}{\partial x} = -\rho\,\frac{\partial^2\psi}{\partial t^2} + 2\Omega\frac{\partial\varphi}{\partial t} + \Omega^2\psi. \tag{12}$$

From equations (9) and (11) one get:

$$\frac{\partial^2\varphi}{\partial x^2} + f_1\,\frac{\partial^2\varphi}{\partial y^2} - f_2\,\frac{\partial\psi}{\partial x} = f_4\left(\frac{\partial^2\varphi}{\partial t^2} + 2\Omega\frac{\partial\psi}{\partial t} - \Omega^2\varphi\right). \tag{13}$$

Where

$$f_1 = \frac{(2\lambda+\alpha+2\mu_L+2\mu_T+P-2\mu_e H_0^2)}{(2\lambda+3\alpha+6\mu_L-2\mu_T+\beta+2P-2\mu_e H_0^2)},$$

$$f_2 = \frac{2\rho g}{(2\lambda+3\alpha+6\mu_L-2\mu_T+\beta+2P-2\mu_e H_0^2)},$$

$$f_3 = \frac{2\rho}{(2\lambda+3\alpha+6\mu_L-2\mu_T+\beta+2P-2\mu_e H_0^2)}.$$

Also, from equations (10) and (12), one gets:

$$\frac{\partial^2\psi}{\partial x^2} + f_4\frac{\partial^2\psi}{\partial y^2} = 0 \qquad (14)$$

where $f_4 = \frac{(\alpha+2\mu_L-2\mu_T+\beta)}{(\alpha+2\mu_L-2\mu_T-P)}$.

The solution for the examined physical variable can be decomposed into normal modes, thereby allowing equations (13) and (14) to be represented in the subsequent form:

$$(\varphi, \psi) = (F(y), G(y))e^{i\omega(x-ct)}, \qquad (15)$$

By denoting b as the wave number in the y-direction and ω as the (complex) time constant, the amplitudes of the field quantities are represented by F(y) and G(y). Upon substituting the values derived from equation (15) into Equation (14), the following expression is obtained:

$$G'' - m^2 G = 0. \qquad (16)$$

The solution for Equation (16) is the form:

$$G = C_1e^{my} + C_2e^{-my}. \qquad (17)$$

In order for Equation (3.3) to accurately represent surface waves and characterize their behavior, it is imperative that it possesses exponential solutions that tend towards infinitesimally small values as $y \to \infty$. i.e. $C_1 = 0$.

$$G = C_2e^{-my} \qquad (18)$$

where, $m^2 = \frac{\omega^2}{f_4}$.

Substituting from Equation (15) into Equation (13), one gets:

$$F'' - f_5\, F - f_6 C_2\, e^{-my} = 0 \tag{19}$$

where

$$f_5 = \frac{\omega^2(1-c^2 f_3)+f_4\Omega^2}{f_1}, \qquad f_6 = \frac{i\,\omega\,(f_{2-2\,c\,f_4\Omega})}{f_1}.$$

From equations (18) and (19), one can obtain the potential function ψ as follow:

$$\psi = C_2\, e^{i\omega(x-ct)-my}, \tag{20}$$

Substituting equations (20), (21), and (8c) into Equation (3) for ψ, u, and v, one gets the following values.

$$\varphi = [C_4\, e^{-f_7 y} + C_6\, e^{-f_8 y} + M\, C_2\, e^{-my}] e^{i\omega(x-ct)}, \tag{21}$$

$$\psi = C_2\, e^{i\omega(x-ct)-my}, \tag{22}$$

where $f_7 = \sqrt{\frac{-d_1\sqrt{{d_1}^2-4\,d_2}}{2}}$, $\quad f_8 = \sqrt{-\frac{d_1}{2} + \frac{1}{2}\sqrt{{d_1}^2 - 4\,d_2}}$, $\quad M = \frac{d_3}{m^4 + d_1\, m^2 - d_2\, m}$ $\quad d_1 = \frac{f_3}{f_2}$, $\quad d_2 = \frac{f_4}{f_2}$, $\quad d_3 = \frac{f_5}{f_2}$.

Substituting equations (20), (21) into Equation (8), one gets the components of displacement as:

$$u = \begin{bmatrix} i\,\omega\, C_4\, e^{-f_7 y} + i\,\omega\, C_6\, e^{-f_8 y} \\ +(i\,\omega\, M - m)\, C_2\, e^{-my} \end{bmatrix} e^{i\omega(x-ct)}, \tag{23}$$

$$v = -\begin{bmatrix} f_7\, C_4\, e^{-f_7 y} + f_8\, C_6\, e^{-f_8 y} \\ +(m\, M + i\,\omega\,)\, C_2\, e^{-my} \end{bmatrix} e^{i\omega(x-ct)}. \tag{24}$$

Substituting from equations (23) and (24) in equations (4)–(7), one obtains:

$$\tau_{xx} = \begin{Bmatrix} \lambda\,({f_4}^2 - \omega^2) + \alpha\,({f_4}^2 - 2\omega^2) - 4\mu_L\omega^2 \\ +\, 2\mu_T\, \omega^2 - \beta\, \omega^2 - \beta_1\,(f_6 {f_4}^2 + f_7) \end{Bmatrix} C_4 e^{-f_4 y} e^{i\omega(x-ct)}$$
$$+ \begin{Bmatrix} \lambda\,({f_6}^2 - \omega^2) + \alpha\,({f_6}^2 - 2\omega^2) - 4\mu_L\omega^2 + \\ 2\mu_T\, \omega^2 \beta\, \omega^2 - \beta_1 (1 - i\,\omega\, \vartheta_0)(f_6 {f_6}^2 + f_7) \end{Bmatrix} C_6 e^{-f_6 y}\, e^{i\omega(x-ct)}$$
$$+ \begin{Bmatrix} \lambda\,(m^2 - \omega^2) + \alpha[(m^2 - 2\omega^2)M - i\omega m] \\ -(4\mu_L - 2\mu_T + \beta) \\ (\omega^2\, M + i\omega\, m) - \beta_1[(m^2 f_6 + f_7)\, M + f_8] \end{Bmatrix} C_2 e^{-my}\, e^{i\omega(x-ct)}$$

$$\tau_{yy} = \begin{Bmatrix} [-\omega^2(\lambda+\alpha+P) - {f_7}^2(\lambda+2\mu_T+P)]C_4\, e^{-f_7 y} + \\ [-\omega^2(\lambda+\alpha+P) - {f_7}^2(\lambda+2\mu_T+P)]\, C_6 e^{-f_8 y} + \\ \begin{bmatrix} -(\omega^2 M + i\,\omega\, m)(\lambda+\alpha+P) - \\ (\lambda+2\mu_T+P)(m^2 M + i\,\omega\, m\,) \end{bmatrix} C_2\, e^{-my} \end{Bmatrix} e^{i\omega(x-ct)}$$

$$\tau_{xy} = \begin{Bmatrix} [-2\,\mu_L\, i\,\omega\, f_7\,]C_4 e^{-f_7 y} + \\ [-2\,\mu_L\, i\,\omega f_8]C_6 e^{-f_7 y} \end{Bmatrix} e^{i\omega(x-ct)}$$
$$+ [-\mu_L(2\, i\,\omega\, m\, M - m^2 - \omega^{\,2})]C_2 e^{-my} e^{i\omega(x-ct)}. \qquad (25)$$

4. Boundary conditions

The stress components and the displacement components at the boundary surface at x = 0 at all times and positions. Mechanical certain boundary conditions when the surface of the half-space undergoes Mechanical load as follows:

$$\tau_{xx} = \tau_{xy} = 0 \quad at \quad x = 0, \mathrm{L} \qquad (26a)$$

$$\tau_{yy} = -\mathrm{p}, \quad at \quad x = 0, L \qquad (26b)$$

Finally, by using the boundary conditions at $x = a\ \ and\ \ at\ \ x = b$, one can determine the arbitrary constants C2, C4, and C6. One can discuss the components of displacement and stresses in rotating fibre-reinforced anisotropic elastic media under effect of and magnetic field an initial stress, from equations (23)-(25) by using computer program. This research study aims to investigate the effects of rotation and an initial magnetic field on the displacement and stress distribution within fiber-reinforced anisotropic elastic media. The researchers utilized a comprehensive analysis approach, employing harmonic vibrations to derive a general solution for determining the displacement and stress components. Furthermore, a specific case within an isotropic generalized elastic medium was examined, taking into account the influence of rotation and an initial magnetic field. To effectively illustrate the obtained results, graphical representations were utilized, providing a visual depiction of the observed phenomena.

5. Numerical results and discussion

In order to present the theoretical findings obtained in this study, compare them within the framework of various elasticity theories, and explore the influence of rotation, magnetic field, gravity, and initial stress in the presence of fiber reinforcement on wave propagation, a numerical application is considered. Through this numerical analysis, all computational results are obtained, providing valuable insights.

The obtained results illustrate the variations in the distribution of important parameters, including the normal stress component, shear stress component, and displacement components u and v. Specifically, the focus is on investigating the impact of these parameters, in the presence of reinforcement, on wave propagation. To elucidate the significance of the findings, numerical results are presented for the relevant physical constants, such as $H_0 = 2x10^2$.

$$\rho = 7800\, kg/m^3, \quad \lambda = 7.59\, Nm^{-2},$$
$$\mu_T = 1.89\, Nm^{-2}, \quad \mu_L = 2.45\, Nm^{-2},$$
$$\alpha = -1.28\, Nm^{-2}, \qquad \beta = 220.90\ Nm^{-2}.$$

These results are visually presented in Figures 1-5, where all variables are expressed in non-dimensional forms. It is worth noting that Figures 1, 2, 3, 4, and 5 depict the changes in field quantities as a function of distance x, based on the classical dynamical elasticity theory, both in the absence and presence of rotation.

Figure 1 and Figure 2 provide valuable insights into the distribution of displacement components u and v as a function of x, showcasing the impact of different rotation values ($\Omega = 0, 1.0, 3.5$) at $H_0 = 2x10^2$. In figure 1 the medium is subject to both magnetic and rotational influences, and the plot elucidates the effect of rotation on the displacement profile. The displacement component *u* initially increases with *x* and then plateaus, indicating a non-linear response of the medium to the rotational forces. As Ω increases, there is a noticeable change in the displacement profile, particularly at higher *x*-values, which reflects the impact of rotational speed on the deformation characteristics of the medium. Figure 2, in particular, focuses on the distribution of displacement component v as a function of x, displaying the influence of different rotation values ($\Omega = 0, 1.0, 3.5$) at $H_0 = 2x10^2$. In the first half of the range, it is observed that the values of the displacement component "u" exhibit an increasing trend, whereas in the latter half of the range, the values display a further increment. Fig. 3,4, and Fig. 5 Illustrate the spatial distribution of stress components, including shear and normal components, as a function of the variable x, considering various rotation values. ($\Omega = 0, 1.0, 3.5$) at $H_0 = 2x10^2$. Fig. 3 Shows the distribution of normal stress component σ_{xy} with respect to x for different values of the rotation ($\Omega = 0, 1.0, 3.5$) at

$H_0 = 2x10^2$. Fig. 4 Shows the distribution of stress component σ_{xy} with respect to x for different values of the rotation ($\Omega = 0, 1.0, 3.5$) at $H_0 = 2x10^2$. Fig. 5 Shows the distribution of shear stress component σ_{xy} with respect to x for different values of the rotation ($\Omega = 0, 1.0, 3.5$) at $H_0 = 2x10^2$.

Figures 4 and 5 demonstrate the varying impact of rotation on the shear and normal components of stresses for different values of x. Figures 3, 4, and 5 display the variations of the normal and shear stress components, namely τ_{xx}, τ_{yy}, and τ_{xy}, against the *x*-coordinate for a fiber-reinforced anisotropic elastic medium exposed to a magnetic field intensity of $H_0=2\times10^7$ A/m, under different rotational speeds (Ω). In Figure 3, the normal stress component τ_{xx} exhibits a non-monotonic behavior with a significant dip and recovery, suggesting a complex interaction between material properties and the applied rotational forces. Figure 4 illustrates the variation of the normal stress component τ_{yy}, which shows a substantial fluctuation, indicating a strong anisotropic response to the applied conditions. Lastly, Figure 5 depicts the variation of the shear stress component τ_{xy}, with a general trend of increasing magnitude with rotation, underscoring the influence of shear deformation under combined magnetic and rotational effects. Collectively, these figures illustrate the intricate stress distribution patterns that arise due to the synergy of magnetic field, rotation, and material anisotropy, which are critical for understanding the mechanical behavior of the medium under study. Specifically, the influence of rotation decreases for lower x values, while it increases for higher x values. It is unfortunate that the current body of knowledge regarding the governing equations of magneto-elastic theory with initial stress is limited. However, the analytical method employed in this study proves highly effective in addressing fiber-reinforcement problems, offering exact solutions within elastic homogenous media without imposing restrictions on physical quantities present in the governing system equations. The observed phenomena discussed in this research hold fundamental significance. The investigation focuses on exploring the effects of rotation, magnetic field, gravity, and initial stress on a fiber-reinforced elastic half-space utilizing the normal mode approach. The obtained results have practical implications for scientists, designers, engineers, and physicists involved in the fields of material science, hyperbolic wave propagation theory, and oil extraction. The findings indicate that the presence of magnetic field, gravity, and initial stress significantly influence the relevant quantities when considering fiber reinforcement and accounting for rotation.

Moreover, the analysis highlights the distinctions between anisotropic and isotropic elastic media, where the deformation of a body is influenced by both the applied forces and the specific boundary conditions [30-33]. The numerical results suggest that as the rotational speed increases, there is a significant alteration in the stress distribution within the medium. This indicates that rotation can greatly influence the internal stress state, which is crucial for applications where rotational dynamics are involved, such as in the design of rotating machinery or aerospace components. Furthermore, the application of a magnetic field introduces additional forces within the material, altering the stress and displacement profiles. This effect is particularly relevant for the development of smart materials and structures that can respond to magnetic fields, providing controlled deformation and stress distribution for advanced engineering applications. The results have practical applications in various fields including mechanical and aerospace engineering, where understanding the behavior of materials under combined mechanical and magnetic influences is vital. For instance, in the design of aerospace structures, where materials are often subjected to complex loading conditions including rotation and magnetic fields, the insights from this research can lead to the development of more resilient materials and structures. Additionally, the study can contribute to the field of materials science, particularly in the development of fiber-reinforced composites with tailored anisotropic properties for specific applications. The analysis of deformation dynamics in fiber-reinforced anisotropic elastic media under the combined effects of magnetic fields, rotation, and gravitational forces provides a critical framework for understanding complex mechanical behaviors in advanced materials. Building upon the concepts of mechanical deformation and microstructural interactions explored in previous studies, such as the shear deformation theories for microtubules [42] and the growth-induced stresses in avascular tumors [43], this research highlights the interplay between external forces and anisotropic properties. Furthermore, the influence of mechanical forces on biological systems, as demonstrated in mosquito biting mechanics [44] and the mechanical behavior of mosquito fascicles [45], illustrates the applicability of these theories to bioengineering and biomaterials. Recent advances in nanoscale modeling, such as the buckling and dynamic behavior of protein microtubules using modified strain gradient theories [37][39], and the effects of thermal stress and surface instability on microtubules [40][41], emphasize the role of external fields in modulating material properties at various scales. This study's findings extend the theoretical groundwork by integrating the effects of initial stresses and gravitational fields, offering a comprehensive model that can be applied to broader scientific and engineering challenges, such as controlling biological systems [46] or optimizing structural designs under multi-physics conditions. The results contribute to the growing body of knowledge on deformation and stress behavior in anisotropic media, paving the way for innovative applications in nanotechnology, biomedical engineering, and material sciences.

6. Conclusion and Recommendations

The problem of investigating the influence of rotation, magnetic field, gravity, and initial stress at the free surface of a fiber-reinforced elastic half-space is examined utilizing the normal mode approach. Through thorough analysis, the following conclusions can be drawn from the findings. • The findings presented in this study hold significant value for scientists engaged in material science, designers of novel materials, engineers, physicists, as well as researchers focusing on the advancement of hyperbolic wave propagation theory and investigations into magnetic field phenomena, with potential applications in improving oil extraction processes. • When considering the effect of rotation in conjunction with the material properties of fiber reinforcement, it is evident that the magnetic field, gravity, and initial stress play a pivotal role in influencing the measured quantities.

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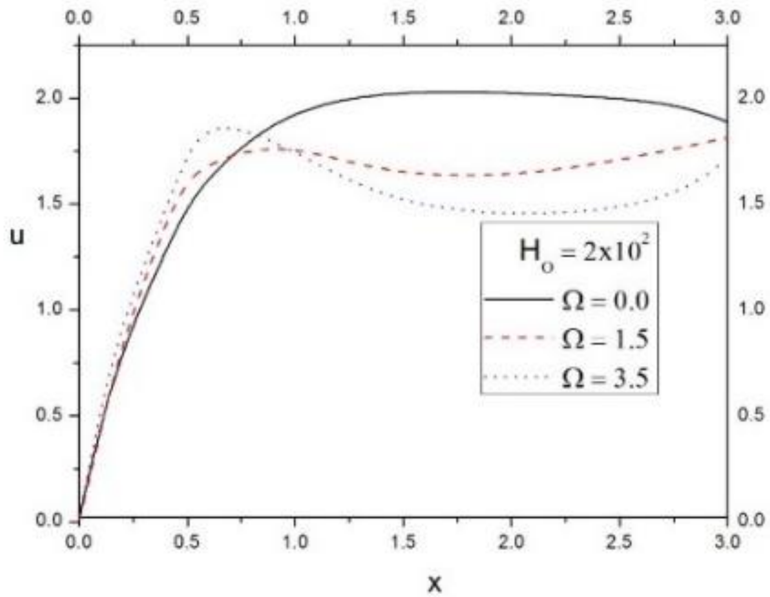

Fig. 1 Shows the graphical representations of the displacement component u in relation to the x-coordinate for various rotational values. ($\Omega = 0, 1.0, 3.5$) at $H_0 = 2x10^2$

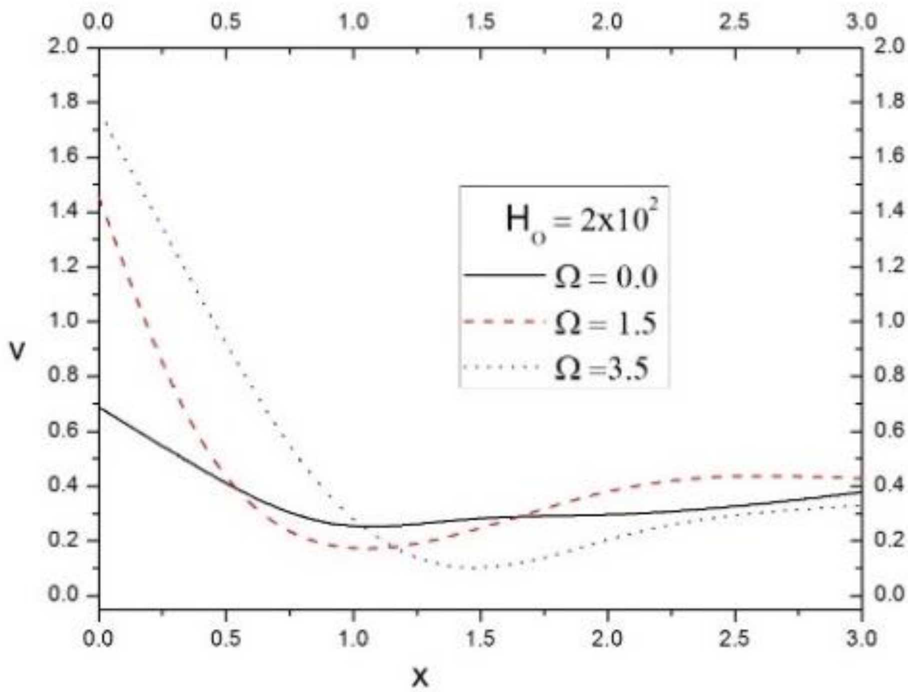

Fig. 2 Shows the graphical representations of the displacement component u in relation to the x-coordinate for various rotational values. ($\Omega = 0, 1.0, 3.5$) at $H_0 = 2x10^2$

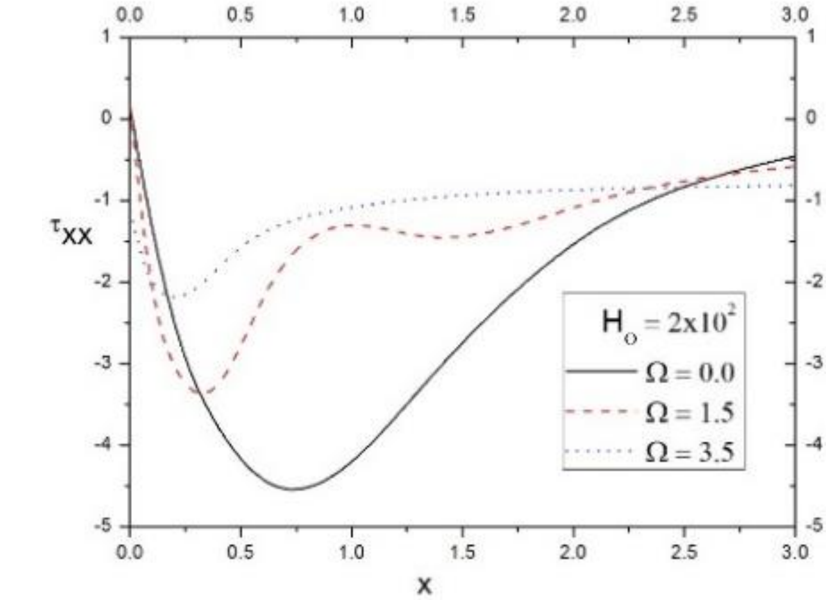

Fig. 3 Shows the variation curves of normal stress component τ_{xx} with respect to x for various rotational values ($\Omega = 0, 1.0, 3.5$) at $H_0 = 2x10^2$

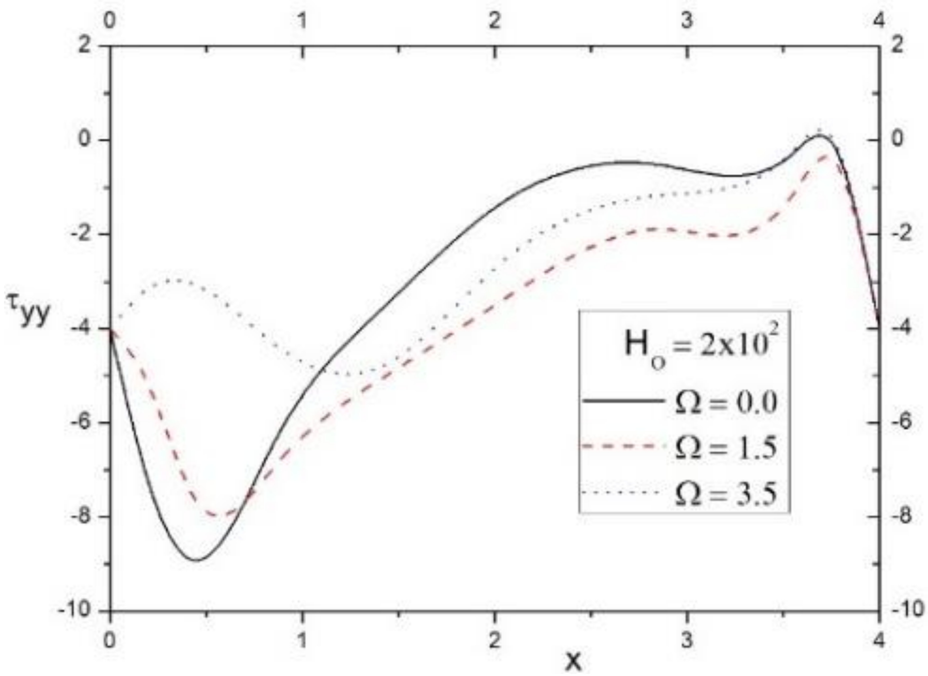

Fig. 4 Shows the variation curves of stress component τ_{yy} with respect to x for various rotational values ($\Omega = 0, 1.0, 3.5$) at $H_0 = 2x10^2$

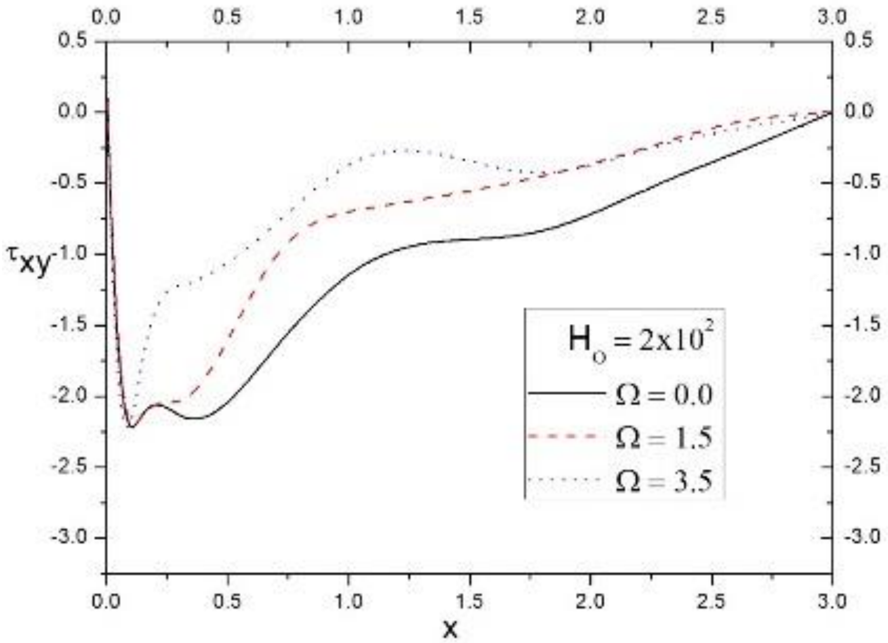

Fig. 5 Shows the variation curves of shear stress component τ_{xy} with respect to x for various rotational values ($\Omega = 0, 1.0, 3.5$) at $H_0 = 2x10^2$

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